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Derivation and Analysis of the Relation between Conductor Sags in Inclined and Levelled Spans Based on Known Data of the Latter

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SUMMARY

The conductor sag is a relevant parameter of the overhead line (OHL) design. Some literatures discuss only the maximum sag, or only the mid–span sag. However, taking into consideration the definition *"The distance measured vertically from a conductor to the straight line (chord) joining its two points of support"*, the sag actually varies in the interval of the span, i.e. increases from zero to maximum then decreases to zero, going from the left support to the right one. It can be appropriately described by the sag equation as a function of x, where x varies from zero to the span length. The actual use of the sag equation is a calculation of the sag at an arbitrary point of the span. It is necessary for example, to obtain the clearance over the conductors at some points which differ from the mid–span.

According to existing literatures the conductor curve is usually drawn in the coordinate system with an origin set at the lowest point of the conductor. In this paper the origin is set at the bottom of the left support and the conductor height is related to the x–axis. Such a classically mathematical approach effectively helps to derive the exact relation between the conductor sags in inclined and levelled spans. Since the developed relation is given as a function of x, it means that the sag at an arbitrary point of an inclined span can be directly calculated from the given sag at the same point of the appropriate levelled span. The derivation and analysis of this relation are the main goals of the recent paper. The usability of the new relation has been proved by practical numerical examples. Also the existing approximate relation between the sags in inclined and levelled spans that can be found in some earlier literatures is adequately discussed. Following a mathematical approach, the equations for the conductor curve, $y(x)$, and the sag, $D(x)$, are given separately both in levelled and inclined spans. These equations are used for computing the conductor height and sag within the span.

The conductor curve is generally considered as a catenary (which is a transcendental function), or in short spans as a parabola (which is an algebraic function). As a logical consequence of this fact, the actual relation is different for the catenary and for the parabola. Therefore, both cases have been discussed in the recent paper. Thus, some special differences between the catenary and the parabola are also recognised and highlighted from the aspect of OHL design. It can be very useful for the electrical network designers.

KEYWORDS

Overhead Lines, Catenary, Parabola, Conductor Curve, Sag Equation, Levelled Span, Inclined Span

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1. INTRODUCTION AND BACKGROUND

Since the towers used for constructing of OHL have usually the same height, levelled spans are commonly present in a flat terrain (see in Figure 1), while inclined spans can be found in a hilly terrain. Inclined spans also occur in a flat terrain when two adjacent towers have different heights. The latter case is shown in Figure 2, where the height difference between the support points is visibly significant. Considering the fact that the support points in levelled spans are on the same elevation, the basic difference between inclined and levelled spans is well seen in these two figures.

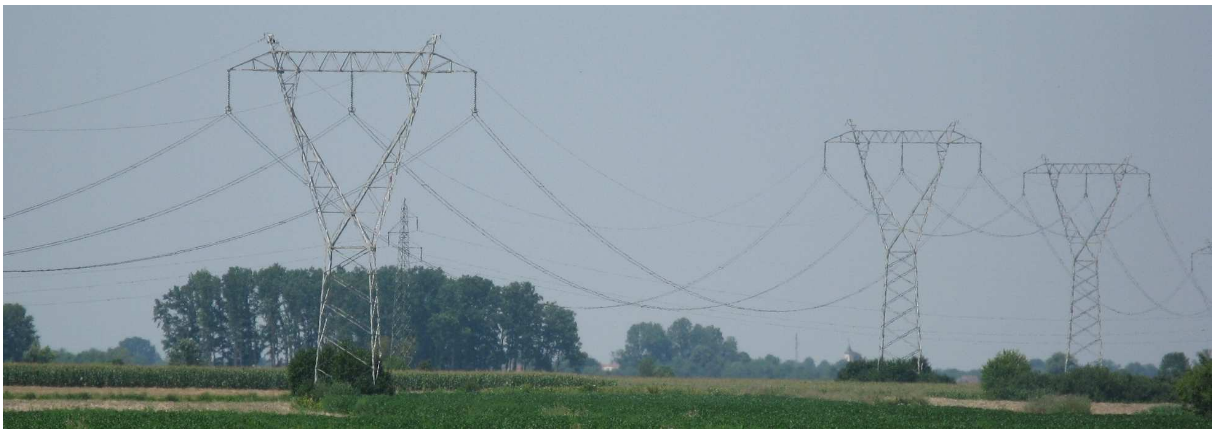

Figure 1. Levelled spans in a flat terrain, a photo taken in Croatia

Figure 2. Inclined span in a flat terrain, a photo taken in Hungary

It is well known that the sag–tension calculation in inclined spans is more difficult than it is in levelled spans. Nowadays the sag–tension tables [1] are available for many different types of conductors, and contain the mid–span sags in levelled spans in dependence of the temperature. It causes a problem that the sag in an inclined span differs from the sag in a levelled span, i.e. $D_{inc}(x) \neq D_{lev}(x)$. This statement concerns not only to a mid–span sag but to the sag at any point of the span excluding the start and end points. That is why the available sag–tension tables cannot be directly applied in inclined spans. Thus, it would be very useful to have an exact relation between the sags in levelled and inclined spans in order to be able to calculate the latter from the first given one. Regarding this, the following statement is often seen in literatures:

"The midpoint sag in inclined span is approximately equal to the sag in a horizontal span equal in length to the inclined span."

This is not an exact relation, but only an approximate one. A further simplification can be found in some earlier literatures, mainly from Europe, which has been adjusted for practical usage. It is given by the relation (1) containing ψ as the angle between a chord and horizontal line:

$$D_{inc}(x) = (1/cos\psi) \cdot D_{lev}(x) \quad \forall \quad 0 \leq x \leq S \tag{1}$$

Defining the span inclination, the angle ψ depends on the vertical distance between the support points. In some literatures (1) is referred only to a parabola, but in some others also to a catenary. However, the newer literatures do not mention the above relation, but gives complicated algorithms for inclined spans [2]. The absence of the exact relation for a quick targeted sag calculation can result in the use of the above approximation. In order to check adequately the usability of (1) and to derive the exact relation between $D_{inc}(x)$ and $D_{lev}(x)$, the mathematical background has been done. As the catenary and the parabola belong to different function groups, naturally the two cases require a separate discussion.

2. CATENARY

In this chapter the conductor curve is considered as a catenary. To obtain the relation between sags in inclined and levelled spans, the following conditions have to be fulfilled:

- Equal catenary constant, c, both in levelled and inclined spans, i.e. $c_{lev} = c_{inc}$,
- Equal span length, S, both in levelled and inclined spans, i.e. $S_{lev} = S_{inc}$.

These conditions practically mean that the conductor curves both in levelled and inclined spans are actually different parts of the same catenary, but the span length is equal in these two cases. The input data for calculations in this chapter are the following: span length, S, heights of the support points related to x–axis, h_1 and h_2, and catenary constant, c. From mathematics it is well known that the latter data determines the shape of the catenary.

2.1 Inclined spans

The equation for the conductor curve in an inclined span can be determined using given input data: S, h_1, h_2, c. All symbols are shown in Figure 3.

Symbols in use:

$A(0;h_1)$ – left–hand side support point
$B(S;h_2)$ – right–hand side support point
$MIN\,(x_{MIN};y_{MIN})$ – catenary's vertex point
S – span length
$y_{inc}(x)$ – conductor curve (catenary)
$y_{line}(x)$ – straight line between the support points
ψ – angle of the span inclination
$D_{inc\,\max}$ – maximum sag in an inclined span

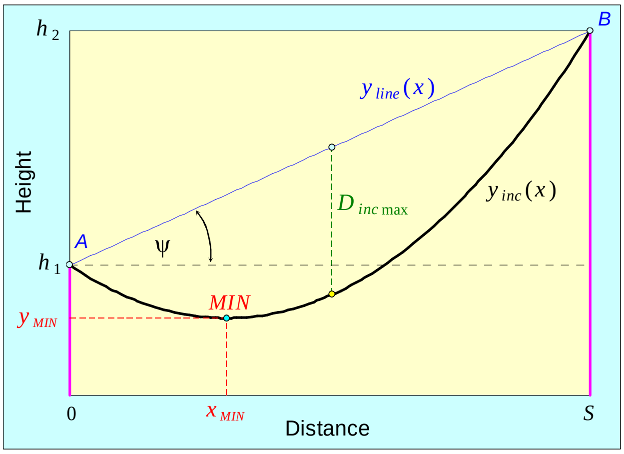

Figure 3. Conductor curve in an inclined span

The basic catenary equation is given by (2) where x_{MIN} is formula (3) taken from CIGRÉ Technical Brochure No. 324 [3] and is expressed by S, h_1, h_2 and c. From condition $y_{inc}(0)=h_1$ also y_{MIN} (5) can be obtained and then the equation for the conductor curve gets its final form (7), in the interval [0,S].

$$y_{inc}(x) = c\cdot ch\frac{x - x_{MIN}}{c} - c + y_{MIN} \qquad (2)$$

$$x_{MIN} = \frac{S}{2} - c\cdot arsh\frac{h_2 - h_1}{2c\cdot sh(S/2c)} \qquad (3)$$

$$y_{inc}(0) = c\cdot ch\frac{-x_{MIN}}{c} - c + y_{MIN} = h_1 \qquad (4)$$

$$y_{MIN} = h_1 - c\cdot ch\frac{x_{MIN}}{c} + c \qquad (5)$$

$$y_{inc}(x) = c\cdot\left(ch\frac{x - x_{MIN}}{c} - ch\frac{x_{MIN}}{c}\right) + h_1 \qquad (6)$$

$$y_{inc}(x) = 2c\cdot sh\frac{x}{2c}\,sh\frac{x - 2x_{MIN}}{2c} + h_1 \qquad (7)$$

Using (7) the conductor height related to x–axis can be calculated at any point of the span. Knowing (7), also the equation for the conductor sag (called as a *sag formula*) can be defined by subtracting (7) from the equation of the straight line, $y_{line}(x)$, which passes through the support points A and B. The result is the sag formula (9), which is usable for the sag calculation at any point of the span.

$$y_{line}(x) = \frac{h_2 - h_1}{S}x + h_1 \qquad (8)$$

$$D_{inc}(x) = y_{line}(x) - y_{inc}(x) = \frac{h_2 - h_1}{S}x - 2c\cdot sh\frac{x}{2c}\,sh\frac{x - 2x_{MIN}}{2c} \qquad (9)$$

2.2 Levelled spans

A levelled span is actually a simplification of an inclined span where the support points are on the same elevation, $h_1=h_2=h$. In this special case the catenary's vertex point (the lowest point) is located at a mid–span, $x_{MIN}=S/2$. For this reason, the equations for the conductor curve (14) and the sag (15) are simpler than the adequate ones in an inclined span. Similarly as it has been shown in the previous section, y_{MIN} (12) is determined from condition $y_{lev}(0)=h$. The conductor curve (catenary) in a levelled span and the used symbols are shown in Figure 4.

$$y_{lev}(x) = c \cdot ch\frac{x - S/2}{c} - c + y_{MIN} \tag{10}$$

$$y_{lev}(0) = c \cdot ch\frac{-S/2}{c} - c + y_{MIN} = h \tag{11}$$

$$y_{MIN} = h - c \cdot ch\frac{S}{2c} + c \tag{12}$$

$$y_{lev}(x) = c \cdot \left(ch\frac{x - S/2}{c} - ch\frac{S}{2c} \right) + h \tag{13}$$

$$y_{lev}(x) = 2c \cdot sh\frac{x}{2c} sh\frac{S - x}{2c} + h \tag{14}$$

$$D_{lev}(x) = h - y_{lev}(x) = 2c \cdot sh\frac{x}{2c} sh\frac{S - x}{2c} \tag{15}$$

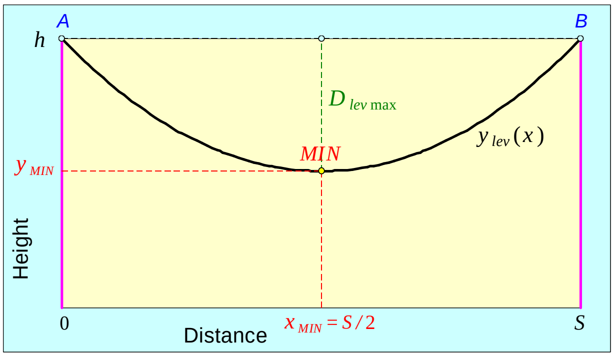

Figure 4. Conductor curve in a levelled span

Naturally, the conductor sag is considered only within the span, i.e. when $0 \le x \le S$, while outside of the span the sag is undefined. The maximum sag in a levelled span is marked as $D_{lev\,\max}$.

2.3 Difference between the sags in inclined and levelled spans

Using the sag formulas from sections 2.1 and 2.2 it is possible to define the difference between the sags in inclined and levelled spans. It is denoted as $\Delta D(x)$ and given by (16):

$$\Delta D(x) = D_{inc}(x) - D_{lev}(x) = \frac{h_2 - h_1}{S} x - 2c \cdot sh\frac{x}{2c} sh\frac{x - 2x_{MIN}}{2c} - 2c \cdot sh\frac{x}{2c} sh\frac{S - x}{2c} \qquad x \in [0, S] \tag{16}$$

Applying mathematical identities for hyperbolic functions, the previous expression can be transformed into the following form:

$$\Delta D(x) = \frac{h_2 - h_1}{S} x - 4c \cdot sh\left(\frac{S}{4c} - \frac{x_{MIN}}{2c} \right) \cdot sh\frac{x}{2c} \cdot ch\left(\frac{S}{4c} - \frac{x - x_{MIN}}{2c} \right) \qquad x \in [0, S] \tag{17}$$

As the function of x, (17) is usable for the calculation of the sag difference at an arbitrary point of the span. Considering (3), $\Delta D(S, h_1, h_2, c, x_{MIN}, x)=\Delta D(S, h_1, h_2, c, x)$. Actually x_{MIN} has been kept instead of (3) in all equations to make them simpler and easier to follow.

Now the graph of (17) can be drawn for analysing e.g. the increase of the span inclination, and then the minimum and the maximum of the sag difference. For this reason, practical numerical examples A and B are presented. The first example has one conductor curve (y_1) in a levelled span and four other ones (y_2, y_3, y_4, y_5) in inclined spans. The height of the left–hand side support point is identical in each of the five spans, but the height of the right–hand side support point is higher in each following span than in the previous one, as the span inclination increases. Both the span length and the catenary constant remain unchanged in all spans. The curve of $\Delta D(x)$ has been drawn in four different cases: $\Delta D_1(x)=D_2(x)–D_1(x)$,….., $\Delta D_4(x)=D_5(x)–D_1(x)$. As it can be seen in Figure 6, $\Delta D(x)$ has three roots (0, r and S) within the span when the span inclination is low, and $\Delta D(x)$ changes sign. As the span inclination increases the root r moves toward the next nearer root, afterwards it does not exist. Then $\Delta D(x)$ does not change sign any more and $D_{inc}(x)>D_{lev}(x)$. It can be observed from all curves in Figure 6 that the value of $|(\Delta D)_{\min}|$, if it exists in the interval (0,S), is not significant at all, while $(\Delta D)_{\max}$ can be significant in the case of high inclination.

Example B is deliberately chosen as a *mirror image* of the example A in order to demonstrate that the method developed is also usable in the opposite case, i.e. when the height of the right–hand side support point is fixed in each span, but the height of the left–hand side one increases. Four curves of $\Delta D(x)$ drawn in this example are the following: $\Delta D_{I}(x)=D_{II}(x)–D_{I}(x)$,….., $\Delta D_{IV}(x)=D_{V}(x)–D_{I}(x)$. The input data for the levelled spans in both examples (A and B) are identical, thus $D_1(x)\equiv D_{I}(x)$.

Example A:

Table 1. Input data in example A

	1	**2**	**3**	**4**	**5**
S [m]	800	800	800	800	800
h_1 [m]	100	100	100	100	100
h_2 [m]	100	160	220	280	340
c [m]	1200	1200	1200	1200	1200

Example B:

Table 2. Input data in example B

	I	**II**	**III**	**IV**	**V**
S [m]	800	800	800	800	800
h_1 [m]	100	160	220	280	340
h_2 [m]	100	100	100	100	100
c [m]	1200	1200	1200	1200	1200

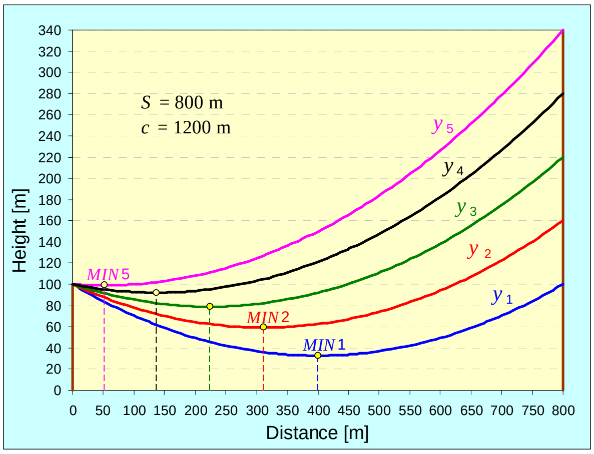

Figure 5. Conductor curves in example A

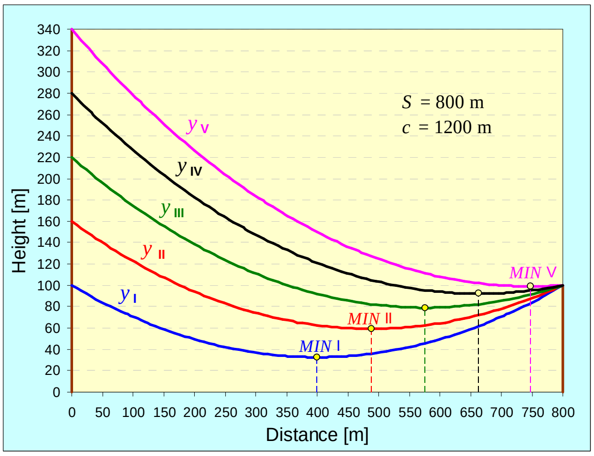

Figure 7. Conductor curves in example B

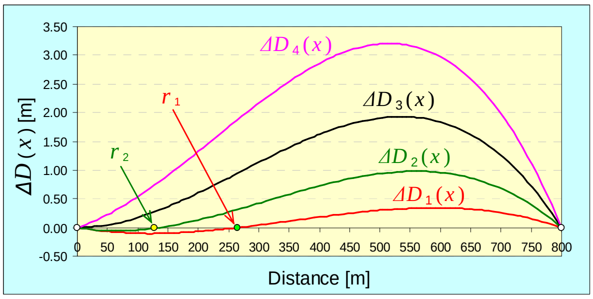

Figure 6. $\Delta D(x)$ curves in example A

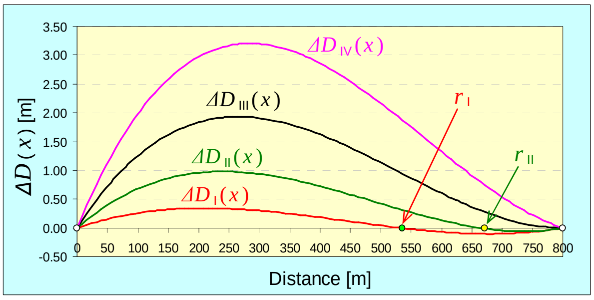

Figure 8. $\Delta D(x)$ curves in example B

Now some important conclusions can be derived from $\Delta D(x)$. First, let us mention the following evident identity:

$$D_{lev}(0) = D_{lev}(S) = D_{inc}(0) = D_{inc}(S) = \Delta D(0) = \Delta D(S) = 0 \tag{18}$$

Thus, $\Delta D(x)$ always has at least two roots, $x_1=0$ and $x_2=S$, in the interval $[0,S]$, but taking into consideration Figure 6 and Figure 8, the third one also exists when the span inclination is not significant. If the third root is denoted as r then the following relevant relations can be written:

Case 1 (three roots):

If $h_1<h_2$ then:

$$\Delta D(x) < 0 \;\Rightarrow\; D_{inc}(x) < D_{lev}(x) \quad \forall \quad x \in (0,r) \tag{19}$$

$$\Delta D(x) > 0 \;\Rightarrow\; D_{inc}(x) > D_{lev}(x) \quad \forall \quad x \in (r,S) \tag{20}$$

$$(\Delta D)_{\max} > \left|(\Delta D)_{\min}\right| \tag{21}$$

If $h_1>h_2$ then:

$$\Delta D(x) > 0 \;\Rightarrow\; D_{inc}(x) > D_{lev}(x) \quad \forall \quad x \in (0,r) \tag{22}$$

$$\Delta D(x) < 0 \;\Rightarrow\; D_{inc}(x) < D_{lev}(x) \quad \forall \quad x \in (r,S) \tag{23}$$

$$(\Delta D)_{\max} > \left|(\Delta D)_{\min}\right| \tag{24}$$

Case 2 (two roots):

If $h_1<h_2$ or $h_1>h_2$ then:

$$\Delta D(x) > 0 \;\Rightarrow\; D_{inc}(x) > D_{lev}(x) \quad \forall \quad x \in (0,S) \tag{25}$$

Returning to *case 1*, it is worth emphasizing that relation $D_{inc}(x)>D_{lev}(x)$ is more dominant than its opposite one. Furthermore, $(\Delta D)_{\max}$ is clearly higher than $|(\Delta D)_{\min}|$, regardless whether $h_1<h_2$ or $h_1>h_2$.

2.4 Application of $\Delta D(x)$

Since equation $\Delta D(x) = D_{inc}(x) - D_{lev}(x)$ can be transformed into (26), the previously determined expression for $\Delta D(x)$ can be practically used for computing $D_{inc}(x)$ from $D_{lev}(x)$ at an arbitrary point x within the span:

$$D_{inc}(x) = D_{lev}(x) + \Delta D(x) \quad \forall \quad x \in [0, S] \tag{26}$$

If S, h, c data for a levelled span are known, then $D_{lev}(x)$ is defined using (15). Considering (3) and (17), $\Delta D(x)$ can be computed from S, c and $\Delta h = h_2 - h_1$, where Δh presents the height difference between the support points of an inclined span. S and c are common data for both spans. Thus, using (26) the sag in an inclined span can be determined from data for a levelled span and a freely chosen Δh for an inclined span. Hence, (26) can be considered as the relation between the catenary sags in inclined and levelled spans. Since all parts in (26) are the functions of x, it can be applied at any point of the span, not only at a mid–span. This justifies the completeness of the shown relation. The applicability of (26) will be presented in a practical numerical task with given $D_{lev}(x)$. The input data for the conductor curves y_1 and y_4 from example A (see table 1 in section 2.3) will be used again.

Task 1:

The given data for the levelled span are the following: S, h, c and D_{lev}(535m); the latter data is a sag at x=535 metres, and $h_1 = h_2 = h$. Calculate the change of the sag if the right–hand side support point is elevated with 180 metres, but S and c data remain unchanged. What is the sag in a formed inclined span at the actual point of the span? Draw the conductor curves and sags on the common diagram.

The data are separated in the following tables:

Table 3. Known data for a levelled span in task 1

S [m]	h_1 [m]	h_2 [m]	c [m]	D_{lev}(535m) [m]
800	100	100	1200	59,68448

Table 4. Known and unknown data for an inclined span in task 1

S [m]	h_1 [m]	h_2 [m]	c [m]	D_{inc}(535m) [m]
800	100	280	1200	?

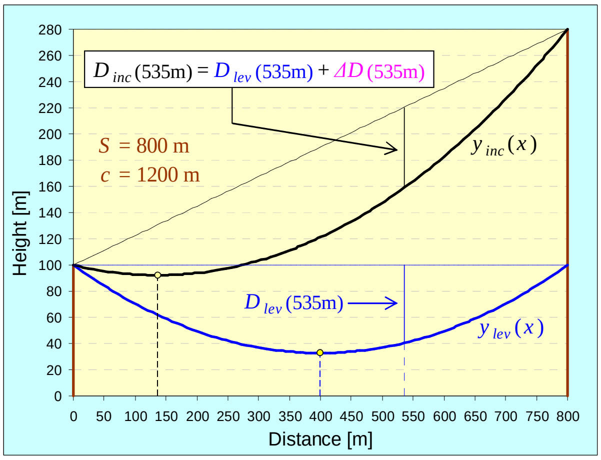

Figure 9. Conductor curves and sags in task 1

Solution and calculations:

Using (3) and (17), ΔD(535m)=1,93104m.

So, D_{inc}(535m) = D_{lev}(535m) + ΔD(535m) = 59,68448m + 1,93104m = 61,61552m

Thus, knowing the sag in a levelled span, the sag in an appropriate inclined span with a freely selected Δh is calculated at an actual point of the span. The sag difference is 1,93104 metres. It is easy to check the correctness of this result with the help of $\Delta D_3(x)$ curve from example A, in Figure 6.

Naturally, the most frequent use of (26) refers to a mid–span sag; in a levelled span it is the maximum sag. In that case (26) gets the following form:

$$D_{inc}(S/2) = D_{lev\max} + \Delta D(S/2) \tag{27}$$

Let us emphasise that the mid–span sag in an inclined span is not denoted as $D_{inc\ \max}$ since the maximum sag of the catenary in non levelled spans is moved from the mid–span [4]. Actually the maximum sag location within the span describes the basic difference between the parabola and the catenary as conductor curves. While the maximum sag of the parabola is always located at the mid–span, i.e. both in levelled and inclined spans, the maximum sag of the catenary in inclined spans is slightly moved from the mid–span toward a higher support point.

Both (26) and (27) can be simply transformed for calculating $D_{lev}(x)$ from given $D_{inc}(x)$, but it is an infrequent task in practice. The conditions listed above ($c_{lev} = c_{inc}$ and $S_{lev} = S_{inc}$) have to be fulfilled in this case as well.

3. PARABOLA AND COMPARISON TO CATENARY

The parabola based calculation is much simpler than the catenary based one since the parabola is a quadratic function, but the catenary is a hyperbolic one. However, their curves can be very similar, and that is why the catenary is frequently approximated by a parabola [5], [6] leading to a significant simplification of the calculation. The input data in this chapter are the following: span length, S, heights of the support points related to x–axis, h_1 and h_2, and maximum sag, $D_{\max}$. The basic parabola equation in a vertex form is given by (28) [7]. Coefficient a determines the shape of the parabola.

$$y = a\left(x - x_{MIN}\right)^2 + y_{MIN} \tag{28}$$

If S, h_1, h_2, $D_{\max}$ are given, then considering (28) and using a coordinate system from Figure 3, the equations for the parabolic conductor curve and the sag in an inclined span can be written as (29) and (30) respectively [8]. Both equations are given in the vertex form.

$$y_{inc}(x) = \frac{4D_{\max}}{S^2}\left[x - \frac{S}{2}\left(1 - \frac{h_2 - h_1}{4D_{\max}}\right)\right]^2 + h_1 - D_{\max}\left(1 - \frac{h_2 - h_1}{4D_{\max}}\right)^2 \qquad x \in [0, S] \tag{29}$$

$$D_{inc}(x) = y_{line}(x) - y_{inc}(x) = \frac{h_2 - h_1}{S}x + h_1 - y_{inc}(x) = \frac{-4D_{\max}}{S^2}\left(x - \frac{S}{2}\right)^2 + D_{\max} \qquad x \in [0, S] \tag{30}$$

According to (30) the sag of the parabolic curve does not depend on Δh (and thus neither on the span inclination), since there are no h_1 and h_2 in the final sag formula. Setting $h_1 = h_2 = h$ in (29) the span becomes levelled. The equations for the conductor curve and the sag are then given by (31) and (32), in the interval $[0,S]$. Note that coefficient $a = (4D_{\max})/S^2$ remains unchanged and thus also $D_{\max}$.

$$y_{lev}(x) = \frac{4D_{\max}}{S^2}\left(x - \frac{S}{2}\right)^2 + h - D_{\max} \tag{31}$$

$$D_{lev}(x) = h - y_{lev}(x) = \frac{-4D_{\max}}{S^2}\left(x - \frac{S}{2}\right)^2 + D_{\max} \tag{32}$$

Since $D_{inc}(x) \equiv D_{lev}(x)$, it means that mathematically there is no difference between the sags of the parabola in inclined and levelled spans and it is valid at each point of the span. On the other hand, the sag of the catenary is dominantly characterised by the relation $D_{inc}(x) > D_{lev}(x)$ in the interval $(0,S)$. Therefore, there is an evident contradiction between the catenary and parabola sags. It is partly compensated in practice by using $1/\cos\psi$ multiplier [9] which increases the sag of the parabola in an inclined span in comparison to the sag in a levelled span. The effect caused is illustrated in Figure 10.

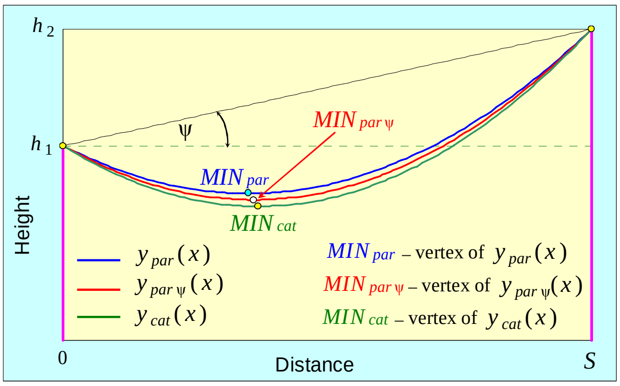

Figure 10. Inclined span with low inclination

Three conductor curves are shown in an inclined span with low inclination. The deepest one is a catenary denoted as $y_{cat}(x)$, the middle one is its parabolic approximation which is modified by using $1/\cos\psi$ and marked as $y_{par\,\psi}(x)$. The highest curve is a classical parabolic approximation of the catenary without using $1/\cos\psi$ and it is denoted as $y_{par}(x)$. The previously mentioned multiplier can be obtained by expression (33).

$$\frac{1}{\cos\psi} = \sqrt{1 + tg^2\psi} = \sqrt{1 + \left(\frac{h_2 - h_1}{S}\right)^2} \tag{33}$$

Notice that Fig. 10 concerns to spans with low inclination only, when the parabola can be used instead of the catenary. However, the following inequality with the catenary and its classical and modified approximations by the parabola is valid for all inclined spans, i.e. both with low and high inclination:

$$\left|y_{par\,\psi}(x) - y_{cat}(x)\right| < y_{par}(x) - y_{cat}(x) \quad \forall \quad 0 < x < S \tag{34}$$

Therefore, the use of $1/\cos\psi$ for a parabola ensures a better approximation of the catenary. According to the above description, the relation between the sags in inclined and levelled spans is different in the case of the classic and modified parabola. It is expressed by (35) and (36):

classic parabola, $y_{par}(x)$

$$D_{inc}(x) = D_{lev}(x) \quad \forall \quad 0 \le x \le S \tag{35}$$

modified parabola, $y_{par\psi}(x)$

$$D_{inc}(x) = \frac{1}{\cos\psi} \cdot D_{lev}(x) \quad \forall \quad 0 \le x \le S \tag{36}$$

4. CONCLUSIONS

As the exact relation between the sags in inclined and levelled spans has been determined both for a catenary and for a parabola, it can be decided whether the use of (1) is recommended or not. The relation (1) is mathematically incorrect for the catenary and its use should be avoided since it generates errors in calculations. Actually (1) produces small errors in spans with low inclination, but for high inclinations it may lead to considerable errors. Since the catenary based calculation is considered as the exact one, the usage of the approximate relation cannot be recommended. The new relation shown in this paper is an exact one.

The situation is very different for usage of (1) in the case of the parabola than that of the catenary. The application of (1) for parabola is recommended, because it decreases the difference between the parabola and the catenary in inclined spans. In this way a modified parabola resembles better the catenary than the classic parabola does.

The following list introduces some specific features of the catenary and its parabolic approximation without and then with the use of $1/cos\psi$. The latter approximation is called here a *modified* one.

	catenary, $y_{cat}(x)$	*classic parabola*, $y_{par}(x)$	*modified parabola*, $y_{par\,\psi}(x)$
1.	$D_{inc\max} > D_{inc}(S/2)$	$D_{inc\max} = D_{inc}(S/2)$	$D_{inc\max} = D_{inc}(S/2)$
2.	$D_{inc}(x) \neq D_{inc}(S-x)$ $0 < x < S/2$	$D_{inc}(x) = D_{inc}(S-x)$ $0 < x < S/2$	$D_{inc}(x) = D_{inc}(S-x)$ $0 < x < S/2$
3.	$\frac{D_{inc}(x)}{D_{lev}(x)} \neq const.$ $0 < x < S$	$\frac{D_{inc}(x)}{D_{lev}(x)} = 1$ $0 < x < S$	$\frac{D_{inc}(x)}{D_{lev}(x)} = \frac{1}{cos\psi}$ $0 < x < S$

According to the expressions written in line 3, it is evident that the function given as a quotient of $D_{inc}(x)$ and $D_{lev}(x)$ in the interval $(0,S)$ is a constant in the case of the parabola (either classic or modified), but it is not in the case of the catenary. This is an important difference between the parabola and the catenary based calculations from the aspect of OHL design.

BIBLIOGRAPHY

[1] D.A. Douglass, R. Thrash: “Sag and Tension of Conductor”, Taylor & Fransis Group LLC., 2006.

[2] J.D. Sakala: “Improved calculation of sag for a conductor supported at unequal heights”, *International Journal of Electrical Engineering Education*, Vol. 45, No. 4, October 2008, pp. 327–335.

[3] CIGRÉ Technical Brochure No. 324, “Sag–Tension Calculation Methods for Overhead Lines”, CIGRÉ WG B2–12, 2007.

[4] A. Hatibović: “Derivation of Equations for Conductor and Sag Curves of an Overhead Line Based on a Given Catenary Constant”, *Periodica Polytechnica Electrical Engineering*, Vol. 58, 2014, DOI 10331/PPee.6993

[5] T.O. Seppa: “Factors Influencing the Accuracy of High Temperature Sag Calculations”, *IEEE Transactions on Power Delivery*, Vol. 9, No. 2, April 1994, pp. 1079–1089.

[6] G. Mirošević, F. Vidaković: “Projektiranje, građenje i održavanje dalekovoda”, KIGEN, Croatia, 2008.

[7] D. Gustafson, P. Frisk, J. Hughes: “College Algebra”, CENGAGE Learning, 2010.

[8] A. Hatibović: “Determination of the Lowest Point of the Conductor in Inclined Spans Based on a Known Maximal Sag of the Parabola”, CIRED 22nd International Conference on Electricity Distribution, Stockholm, Sweden, June 2013, Paper No. 0150.

[9] A. Hatibović: “Analysis of the Quadratic Equation for the Conductor Sag Based on a Given Maximal Sag”, *Bosanskohercegovačka elektrotehnika*, No. 7, BH K CIGRÉ, 2013.